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Chapter 1

Resolutions of the ground ring

TietzeReducedResolution(R) Inputs a $\mathbb{Z}G$ -resolution R and returns a $\mathbb{Z}G$ -resolution S which is obtained from R by
 ResolutionArithmeticGroup("PSL(4,Z)", n) Inputs a positive integer n and one of the following strings:

"SL(2,Z)", "SL(3,Z)", "PGL(3,Z[i])", "PGL(3,Eisenstein_Integers)", "PSL(4,Z)", "PSL(4,Z)_b", "PSL(4,Z)_c",

or one of the following strings

"SL(2,Z[sqrt(-2)])", "SL(2,Z[sqrt(-7)])", "SL(2,Z[sqrt(-11)])", "SL(2,Z[sqrt(-19)])", "SL(2,Z[sqrt(-43)])", "SL(2,Z[sqrt(-47)])",

It returns n terms of a free ZG -resolution for the group G described by the string. (Subscripts $_b$, $_c$, $_d$ denote alternate

Data for the first list of resolutions was provided provided by MATHIEU DUTOUR. Data for the second list was provided by

FreeGResolution(P , n) FreeGResolution(P , n , p) Inputs a non-free ZG -resolution P with finite stabilizer group G and integer n and p .

ResolutionGTree(P , n) Inputs a non-free ZG -resolution P of dimension 1 (i.e. a G -tree) with finite stabilizer group G and integer n .

ResolutionSL2Z(p , n) Inputs positive integers m, n and returns n terms of a ZG -resolution for the group $G = SL(2, \mathbb{Z}[1/m])$.

ResolutionAbelianGroup(L , n) ResolutionAbelianGroup(G , n) Inputs a list $L := [m_1, m_2, \dots, m_d]$ of nonnegative integers and integer n .

ResolutionAlmostCrystalGroup(G , n) Inputs a positive integer n and an almost crystallographic pcg group G . It returns n terms of a free ZG -resolution.

ResolutionAlmostCrystalQuotient(G , n , c) ResolutionAlmostCrystalQuotient(G , n , c , $false$) An almost crystallographic pcg group G and integer n and c .

ResolutionArtinGroup(D , n) Inputs a Coxeter diagram D and an integer $n > 1$. It returns n terms of a free ZG -resolution for the Artin group of D .

ResolutionAsphericalPresentation(F , R , n) Inputs a free group F , a set R of words in F which constitute an aspherical presentation of a group G and integer n .

ResolutionBieberbachGroup(G) ResolutionBieberbachGroup(G , v) Inputs a torsion free crystallographic group G and integer v .

ResolutionCoxeterGroup(D , n) Inputs a Coxeter diagram D and an integer $n > 1$. It returns k terms of a free ZG -resolution for the Coxeter group of D .

ResolutionDirectProduct(R , S) ResolutionDirectProduct(R , S , "internal") Inputs a ZG -resolution R and a ZH -resolution S .

ResolutionExtension(g , R , S) ResolutionExtension(g , R , S , "TestFiniteness") ResolutionExtension(g , R , S , "TestFiniteness", n)

ResolutionFiniteDirectProduct(R , S) ResolutionFiniteDirectProduct(R , S , "internal") Inputs a ZG -resolution R and a ZH -resolution S .

ResolutionFiniteExtension($gensE$, $gensG$, R , n) ResolutionFiniteExtension($gensE$, $gensG$, R , n , $true$)

ResolutionFiniteGroup($gens$, n) ResolutionFiniteGroup($gens$, n , $true$) ResolutionFiniteGroup($gens$, n , $false$)

ResolutionFiniteSubgroup(R , K) ResolutionFiniteSubgroup(R , $gensG$, $gensK$) Inputs a ZG -resolution for a group G and a subgroup K of finite index.

ResolutionGraphOfGroups(D , n) ResolutionGraphOfGroups(D , n , L) Inputs a graph of groups D and a positive integer n .

ResolutionNilpotentGroup(G , n) ResolutionNilpotentGroup(G , n , "TestFiniteness") Inputs a nilpotent group G and integer n .

ResolutionNormalSeries(L , n) ResolutionNormalSeries(L , n , $true$) ResolutionNormalSeries(L , n , $false$)

ResolutionPrimePowerGroup(P , n) ResolutionPrimePowerGroup(G , n , p) Inputs a p -group P and integer $n > 0$.

ResolutionSmallFpGroup(G , n) ResolutionSmallFpGroup(G , n , p) Inputs a small finitely presented group G and integer n .

ResolutionSubgroup(R , K) Inputs a ZG -resolution for an (infinite) group G and a subgroup K of finite index $|G : K|$.

ResolutionSubnormalSeries(L , n) Inputs a positive integer n and a list $L = [L_1, \dots, L_k]$ of subgroups L_i of a finite group G .

TwistedTensorProduct(R , S , $EhomG$, $GmapE$, $NhomE$, $NEhomN$, $EltSE$, $Mult$, $InvE$) Inputs a ZG -resolution R , a ZH -resolution S , and a ZG -resolution E .

ConjugatedResolution(R , x) Inputs a ZG -resoluton R and an element x from some group containing G . It returns a resolution for the conjugate group.

RecalculateIncidenceNumbers(R) Inputs a ZG -resoluton R which arises as the cellular chain complex of a regular CW-complex.

Chapter 2

Resolutions of modules

| `ResolutionFpModule(M, n)` Inputs an FpG -module M and a positive integer n . It returns n terms of a minimal free

Chapter 3

Induced equivariant chain maps

| `EquivariantChainMap(R, S, f)` Inputs a ZG -resolution R , a ZG' -resolution S , and a group homomorphism $f : G \rightarrow G'$

Chapter 4

Functors

ExtendScalars($R, G, EltsG$) Inputs a ZH -resolution R , a group G containing H as a subgroup, and a list $EltsG$ of
HomToIntegers(X) Inputs either a ZG -resolution $X = R$, or an equivariant chain map $X = (F : R \rightarrow S)$. It returns
HomToIntegersModP(R) Inputs a ZG -resolution R and returns the cochain complex obtained by applying $HomZG$
HomToIntegralModule(R, f) Inputs a ZG -resolution R and a group homomorphism $f : G \rightarrow GL_n(Z)$ to the group
TensorWithIntegralModule(R, f) Inputs a ZG -resolution R and a group homomorphism $f : G \rightarrow GL_n(Z)$ to the group
HomToGModule(R, A) Inputs a ZG -resolution R and an abelian G -outer group A . It returns the G -cocomplex obtained
InduceScalars(R, hom) Inputs a ZQ -resolution R and a surjective group homomorphism $hom : G \rightarrow Q$. It returns
LowerCentralSeriesLieAlgebra(G) LowerCentralSeriesLieAlgebra(f) Inputs a pcp group G . If each qu
TensorWithIntegers(X) Inputs either a ZG -resolution $X = R$, or an equivariant chain map $X = (F : R \rightarrow S)$. It
FilteredTensorWithIntegers(R) Inputs a ZG -resolution R for which "filteredDimension" lies in NamesOfCom
TensorWithTwistedIntegers(X, rho) Inputs either a ZG -resolution $X = R$, or an equivariant chain map $X = (F$
TensorWithIntegersModP(X, p) Inputs either a ZG -resolution $X = R$, or a characteristics 0 chain complex, or an
TensorWithTwistedIntegersModP(X, p, rho) Inputs either a ZG -resolution $X = R$, or an equivariant chain map X
TensorWithRationals(R) Inputs a ZG -resolution R and returns the chain complex obtained by tensoring with the

Chapter 5

Chain complexes

`ChainComplex(T)` Inputs a pure cubical complex, or cubical complex, or simplicial complex T and returns the (often)

`ChainComplexOfPair(T, S)` Inputs a pure cubical complex or cubical complex T and contractible subcomplex S . It

`ChevalleyEilenbergComplex(X, n)` Inputs either a Lie algebra $X = A$ (over the ring of integers Z or over a field

`LeibnizComplex(X, n)` Inputs either a Lie or Leibniz algebra $X = A$ (over the ring of integers Z or over a field K

`SuspendedChainComplex(C)` Inputs a chain complex C and returns the chain complex S defined by applying the de

`ReducedSuspendedChainComplex(C)` Inputs a chain complex C and returns the chain complex S defined by applyi

`CoreducedChainComplex(C)` `CoreducedChainComplex(C, 2)` Inputs a chain complex C and returns a quasi-isom

`TensorProductOfChainComplexes(C, D)` Inputs two chain complexes C and D of the same characteristic and retur

`LefschetzNumber(F)` Inputs a chain map $F: C \rightarrow C$ with common source and target. It returns the Lefschetz numbe

Chapter 6

Homology and cohomology groups

`Cohomology(X,n)` Inputs either a cochain complex $X = C$ (or G -cocomplex C) or a cochain map $X = (C \rightarrow D)$ in
`CohomologyModule(C,n)` Inputs a G -cocomplex C together with a non-negative integer n . It returns the cohomology
`CohomologyPrimePart(C,n,p)` Inputs a cochain complex C in characteristic 0, a positive integer n , and a prime p .
`GroupCohomology(X,n)` `GroupCohomology(X,n,p)` Inputs a positive integer n and either a finite group $X = G$ or a nilpotent
`GroupHomology(X,n)` `GroupHomology(X,n,p)` Inputs a positive integer n and either a finite group $X = G$ or a nilpotent
`PersistentHomologyOfQuotientGroupSeries(S,n)` `PersistentHomologyOfQuotientGroupSeries(S,n,p)`
`PersistentCohomologyOfQuotientGroupSeries(S,n)` `PersistentCohomologyOfQuotientGroupSeries(S,n,p)`
`UniversalBarCode("UpperCentralSeries",n,d)` `UniversalBarCode("UpperCentralSeries",n,d,k)` Inputs
`PersistentHomologyOfSubGroupSeries(S,n)` `PersistentHomologyOfSubGroupSeries(S,n,p,Resolution)`
`PersistentHomologyOfFilteredChainComplex(C,n,p)` Inputs a filtered chain complex C (of characteristic 0) and a prime p .
`PersistentHomologyOfCommutativeDiagramOfPGroups(D,n)` Inputs a commutative diagram D of finite p -groups.
`PersistentHomologyOfPureCubicalComplex(L,n,p)` Inputs a positive integer n , a prime p and an increasing chain of subgroups
`ZZPersistentHomologyOfPureCubicalComplex(L,n,p)` Inputs a positive integer n , a prime p and any sequence of subgroups
`RipsHomology(G,n)` `RipsHomology(G,n,p)` Inputs a graph G , a non-negative integer n (and optionally a prime number p).
`BarCode(P)` Inputs an integer persistence matrix P and returns the same information in the form of a binary matrix (0/1).
`BarCodeDisplay(P)` `BarCodeDisplay(P,"mozilla")` Inputs an integer persistence matrix P , and an optional string for the browser.
`Homology(X,n)` Inputs either a chain complex $X = C$ or a chain map $X = (C \rightarrow D)$. If $X = C$ then the torsion coefficient is
`HomologyPb(C,n)` This is a back-up function which might work in some instances where `Homology(C,n)` fails. It is a
`HomologyVectorSpace(X,n)` Inputs either a chain complex $X = C$ or a chain map $X = (C \rightarrow D)$ in prime characteristic
`HomologyPrimePart(C,n,p)` Inputs a chain complex C in characteristic 0, a positive integer n , and a prime p . It returns
`LeibnizAlgebraHomology(A,n)` Inputs a Lie or Leibniz algebra $X = A$ (over the ring of integers Z or over a field K) and a positive integer n .
`LieAlgebraHomology(A,n)` Inputs a Lie algebra A (over the integers or a field) and a positive integer n . It returns the
`PrimePartDerivedFunctor(G,R,F,n)` Inputs a finite group G , a positive integer n , at least $n+1$ terms of a ZP -resolution
`RankHomologyPGroup(G,n)` `RankHomologyPGroup(R,n)` `RankHomologyPGroup(G,n,"empirical")` Inputs a (smallish) p -group
`RankPrimeHomology(G,n)` Inputs a (smallish) p -group G together with a positive integer n . It returns a function `dim`

Chapter 7

Poincare series

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`EfficientNormalSubgroups(G)` `EfficientNormalSubgroups(G,k)` Inputs a prime-power group G and, optionally, a prime k .

`ExpansionOfRationalFunction(f,n)` Inputs a positive integer n and a rational function $f(x) = p(x)/q(x)$ where $p(x)$ and $q(x)$ are polynomials with integer coefficients.

`PoincareSeries(G,n)` `PoincareSeries(R,n)` `PoincareSeries(L,n)` `PoincareSeries(G)` Inputs a finite group G , a ring R , a lattice L , or a finite group G , and a positive integer n . It returns a quotient of polynomials $f(x) = P(x)/Q(x)$ whose denominator $Q(x)$ is a product of terms $(1 - x^g)$ for g in G .

`PoincareSeriesPrimePart(G,p,n)` Inputs a finite group G , a prime p , and a positive integer n . It returns a quotient of polynomials $f(x) = P(x)/Q(x)$ whose denominator $Q(x)$ is a product of terms $(1 - x^g)$ for g in G .

`Prank(G)` Inputs a p -group G and returns the rank of the largest elementary abelian subgroup.

Chapter 8

Cohomology ring structure

`IntegralCupProduct(R,u,v,p,q)` `IntegralCupProduct(R,u,v,p,q,P,Q,N)` (Various functions used to compute cup products)
`IntegralRingGenerators(R,n)` Inputs at least $n+1$ terms of a ZG -resolution and integer $n > 0$. It returns a minimal set of generators for the integral cohomology ring.
`ModPCohomologyGenerators(G,n)` `ModPCohomologyGenerators(R)` Inputs either a p -group G and positive integer n , or a ring R .
`ModPCohomologyRing(G,n)` `ModPCohomologyRing(G,n,level)` `ModPCohomologyRing(R)` `ModPCohomologyRing(R,n)`
`ModPRingGenerators(A)` Inputs a mod p cohomology ring A (created using the preceding function). It returns a minimal set of generators for the integral cohomology ring.
`Mod2CohomologyRingPresentation(G)` `Mod2CohomologyRingPresentation(G,n)` `Mod2CohomologyRingPresentation(R)`

Chapter 9

Cohomology rings of p -groups (mainly $p = 2$)

The functions on this page were written by PAUL SMITH. (They are included in HAP but they are also independently included in Paul Smiths HAPprime package.)

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| `Mod2CohomologyRingPresentation(G)` `Mod2CohomologyRingPresentation(G,n)` `Mod2CohomologyRingP`
| `PoincareSeriesLHS(G)` Inputs a finite 2-group G and returns a quotient of polynomials $f(x) = P(x)/Q(x)$ whose

Chapter 10

Commutator and nonabelian tensor computations

`BaerInvariant(G,c)` Inputs a nilpotent group G and integer $c > 0$. It returns the Baer invariant $M^{(c)}(G)$ defined as
`Coclass(G)` Inputs a group G of prime-power order p^n and nilpotency class c say. It returns the integer $r = n - c$.
`EpiCentre(G,N)` `EpiCentre(G)` Inputs a finite group G and normal subgroup N and returns a subgroup $Z^*(G,N)$
`NonabelianExteriorProduct(G,N)` Inputs a finite group G and normal subgroup N . It returns a record E with the f
`NonabelianSymmetricKernel(G)` `NonabelianSymmetricKernel(G,m)` Inputs a finite or nilpotent infinite gro
`NonabelianSymmetricSquare(G)` `NonabelianSymmetricSquare(G,m)` Inputs a finite or nilpotent infinite gro
`NonabelianTensorProduct(G,N)` Inputs a finite group G and normal subgroup N . It returns a record E with the f
`NonabelianTensorSquare(G)` `NonabelianTensorSquare(G,m)` Inputs a finite or nilpotent infinite group G an
`RelativeSchurMultiplier(G,N)` Inputs a finite group G and normal subgroup N . It returns the homology group
`TensorCentre(G)` Inputs a group G and returns the largest central subgroup N such that the induced homomorphis
`ThirdHomotopyGroupOfSuspensionB(G)` `ThirdHomotopyGroupOfSuspensionB(G,m)` Inputs a finite or nilpo
`UpperEpicentralSeries(G,c)` Inputs a nilpotent group G and an integer c . It returns the c -th term of the upper e

Chapter 11

Lie commutators and nonabelian Lie tensors

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Functions on this page are joint work with HAMID MOHAMMADZADEH, and implemented by him.

`LieCoveringHomomorphism(L)` Inputs a finite dimensional Lie algebra L over a field, and returns a surjective Lie h

`LeibnizQuasiCoveringHomomorphism(L)` Inputs a finite dimensional Lie algebra L over a field, and returns a surj

`LieEpiCentre(L)` Inputs a finite dimensional Lie algebra L over a field, and returns an ideal $Z^*(L)$ of the centre of L .

`LieExteriorSquare(L)` Inputs a finite dimensional Lie algebra L over a field. It returns a record E with the follow

`LieTensorSquare(L)` Inputs a finite dimensional Lie algebra L over a field and returns a record T with the follow

`LieTensorCentre(L)` Inputs a finite dimensional Lie algebra L over a field and returns the largest ideal N such tha

Chapter 12

Generators and relators of groups

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`CayleyGraphOfGroupDisplay(G,X)` `CayleyGraphOfGroupDisplay(G,X,"mozilla")` Inputs a finite group G
`IdentityAmongRelatorsDisplay(R,n)` `IdentityAmongRelatorsDisplay(R,n,"mozilla")` Inputs a free Z -
`IsAspherical(F,R)` Inputs a free group F and a set R of words in F . It performs a test on the 2-dimensional CW-
`PresentationOfResolution(R)` Inputs at least two terms of a reduced ZG -resolution R and returns a record P with
`TorsionGeneratorsAbelianGroup(G)` Inputs an abelian group G and returns a generating set $[x_1, \dots, x_n]$ where n

Chapter 14

Cocycles

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`CcGroup(A, f)` Inputs a G -module A (i.e. an abelian G -outer group) and a standard 2-cocycle $f: G \times G \rightarrow A$. It returns a G -module C and a standard 2-cocycle f_C such that (C, f_C) is a cocycle condition for A .
`CocycleCondition(R, n)` Inputs a resolution R and an integer $n > 0$. It returns an integer matrix M with the following property: M is a cocycle condition for R of order n .
`StandardCocycle(R, f, n)` Inputs a resolution R and a standard 2-cocycle $f: G \times G \rightarrow A$. It returns a standard 2-cocycle f_C such that (R, f_C) is a cocycle condition for A .
`StandardCocycle(R, f, n, q)` Inputs a ZG -resolution R (with contracting homotopy), a positive integer n and an integer q . It returns a standard 2-cocycle f_C such that (R, f_C) is a cocycle condition for A .
`Syzygy(R, g)` Inputs a ZG -resolution R (with contracting homotopy) and a list $g = [g[1], \dots, g[n]]$ of elements in G . It returns a list of syzygies s such that s is a cocycle condition for R of order n .

Chapter 15

Words in free ZG -modules

`AddFreeWords(v,w)` Inputs two words v,w in a free ZG -module and returns their sum $v+w$. If the characteristic of Z is p , it returns the sum modulo p .

`AddFreeWordsModP(v,w,p)` Inputs two words v,w in a free ZG -module and the characteristic p of Z . It returns the sum modulo p .

`AlgebraicReduction(w)` Inputs a word w in a free ZG -module and returns a reduced version of the word in which no two adjacent letters are inverses.

`AlgebraicReduction(w,p)` Inputs a word w in a free ZG -module and returns a reduced version of the word in which no two adjacent letters are inverses, and the sum of the exponents of each letter is a multiple of p .

`Multiply Word(n,w)` Inputs a word w and integer n . It returns the scalar multiple $n \cdot w$.

`Negate([i,j])` Inputs a pair $[i,j]$ of integers and returns $[-i,j]$.

`NegateWord(w)` Inputs a word w in a free ZG -module and returns the negated word $-w$.

`PrintZGword(w,elts)` Inputs a word w in a free ZG -module and a (possibly partial but sufficient) listing `elts` of the elements of G . It prints the word w in terms of the elements of G .

`TietzeReduction(S,w)` Inputs a set S of words in a free ZG -module, and a word w in the module. The function returns a list of words in S whose sum is w .

`ResolutionBoundaryOfWord(R,n,w)` Inputs a resolution R , a positive integer n and a list w representing a word in the free ZG -module. It returns a list of words in R whose sum is w .

Chapter 16

FpG-modules

`CompositionSeriesOfFpGModules(M)` Inputs an *FpG*-module M and returns a list of *FpG*-modules that constitute a composition series for M .

`DirectSumOfFpGModules(M,N)` `DirectSumOfFpGModules([M[1], M[2], ..., M[k]])` Inputs two *FpG*-modules M and N or a list of *FpG*-modules $M[1], M[2], \dots, M[k]$.

`FpGModule(A,P)` `FpGModule(A,G,p)` Inputs a p -group P and a matrix A whose rows have length a multiple of the order of P , or a p -group G and a matrix A whose rows have length a multiple of the order of G .

`FpGModuleDualBasis(M)` Inputs an *FpG*-module M . It returns a record R with two components: $R.freeModule$ is a free *FpG*-module F and $R.basis$ is a list of elements of M that form a dual basis for M .

`FpGModuleHomomorphism(M,N,A)` `FpGModuleHomomorphismNC(M,N,A)` Inputs *FpG*-modules M and N over a common p -group G and a matrix A whose rows have length a multiple of the order of G .

`DesuspensionFpGModule(M,n)` `DesuspensionFpGModule(R,n)` Inputs a positive integer n and an *FpG*-module M or a record R with a component $R.freeModule$ which is a free *FpG*-module F .

`RadicalOfFpGModule(M)` Inputs an *FpG*-module M with G a p -group, and returns the Radical of M as an *FpG*-module.

`RadicalSeriesOfFpGModule(M)` Inputs an *FpG*-module M and returns a list of *FpG*-modules that constitute the radical series for M .

`GeneratorsOfFpGModule(M)` Inputs an *FpG*-module M and returns a matrix whose rows correspond to a minimal generating set for M .

`ImageOfFpGModuleHomomorphism(f)` Inputs an *FpG*-module homomorphism $f : M \rightarrow N$ and returns its image as an *FpG*-module.

`GroupAlgebraAsFpGModule(G)` Inputs a p -group G and returns its mod p group algebra as an *FpG*-module.

`IntersectionOfFpGModules(M,N)` Inputs two *FpG*-modules M, N arising as submodules in a common free module F .

`IsFpGModuleHomomorphismData(M,N,A)` Inputs *FpG*-modules M and N over a common p -group G . Also inputs a matrix A whose rows have length a multiple of the order of G .

`MaximalSubmoduleOfFpGModule(M)` Inputs an *FpG*-module M and returns one maximal *FpG*-submodule of M .

`MaximalSubmodulesOfFpGModule(M)` Inputs an *FpG*-module M and returns the list of maximal *FpG*-submodules of M .

`MultipleOfFpGModule(w,M)` Inputs an *FpG*-module M and a list $w := [g_1, \dots, g_t]$ of elements in the group $G = M$.

`ProjectedFpGModule(M,k)` Inputs an *FpG*-module M of ambient dimension $n|G|$, and an integer k between 1 and n .

`RandomHomomorphismOfFpGModules(M,N)` Inputs two *FpG*-modules M and N over a common group G . It returns a random *FpG*-module homomorphism $f : M \rightarrow N$.

`Rank(f)` Inputs an *FpG*-module homomorphism $f : M \rightarrow N$ and returns the dimension of the image of f as a vector space over \mathbb{F}_p .

`SumOfFpGModules(M,N)` Inputs two *FpG*-modules M, N arising as submodules in a common free module $(FG)^n$.

`SumOp(f,g)` Inputs two *FpG*-module homomorphisms $f, g : M \rightarrow N$ with common source and common target. It returns the sum $f + g$.

`VectorsToFpGModuleWords(M,L)` Inputs an *FpG*-module M and a list $L = [v_1, \dots, v_k]$ of vectors in M . It returns a list of words in the generators of M that represent the vectors in L .

Chapter 17

Meataxe modules

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| `DesuspensionMtxModule(M)` Inputs a meataxe module M over the field of p elements and returns an FpG-module.
| `FpG_to_MtxModule(M)` Inputs an FpG-module M and returns an isomorphic meataxe module.
| `GeneratorsOfMtxModule(M)` Inputs a meataxe module M acting on, say, the vector space V . The function returns a

Chapter 18

G-Outer Groups

`GOuterGroup(E,N)` `GOuterGroup()` Inputs a group E and normal subgroup N . It returns N as a G -outer group where $G = E/N$.

`GOuterGroupHomomorphismNC(A,B,phi)` `GOuterGroupHomomorphismNC()` Inputs G -outer groups A and B with common acting group G , and a group homomorphism $\phi: G \rightarrow G$.

`GOuterHomomorphismTester(A,B,phi)` Inputs G -outer groups A and B with common acting group G , and a group homomorphism $\phi: G \rightarrow G$.

`Centre(A)` Inputs G -outer group A and returns the group theoretic centre of $\text{ActedGroup}(A)$ as a G -outer group.

`DirectProductGog(A,B)` `DirectProductGog(Lst)` Inputs G -outer groups A and B with common acting group G , and a list Lst of G -outer groups.

Chapter 19

Cat-1-groups

`AutomorphismGroupAsCatOneGroup(G)` Inputs a group G and returns the Cat-1-group C corresponding to the cross

`HomotopyGroup(C, n)` Inputs a cat-1-group C and an integer n . It returns the n th homotopy group of C .

`HomotopyModule(C, 2)` Inputs a cat-1-group C and an integer $n=2$. It returns the second homotopy group of C as a G -

`QuasiIsomorph(C)` Inputs a cat-1-group C and returns a cat-1-group D for which there exists some homomorphism

`ModuleAsCatOneGroup(G, alpha, M)` Inputs a group G , an abelian group M and a homomorphism $\alpha: G \rightarrow \text{Aut}(M)$.

`MooreComplex(C)` Inputs a cat-1-group C and returns its Moore complex as a G -complex (i.e. as a complex of group

`NormalSubgroupAsCatOneGroup(G, N)` Inputs a group G with normal subgroup N . It returns the Cat-1-group C cor

`XmodToHAP(C)` Inputs a cat-1-group C obtained from the Xmod package and returns a cat-1-group D for which `IsHa`

Chapter 20

Simplicial groups

`NerveOfCatOneGroup(G, n)` Inputs a cat-1-group G and a positive integer n . It returns the low-dimensional part of f

This function applies both to cat-1-groups for which `IsHapCatOneGroup(G)` is true, and to cat-1-groups produced us

This function was implemented by VAN LUYEN LE.

`EilenbergMacLaneSimplicialGroup(G, n, dim)` Inputs a group G , a positive integer n , and a positive integer dim

This function was implemented by VAN LUYEN LE.

`EilenbergMacLaneSimplicialGroupMap(f, n, dim)` Inputs a group homomorphism $f : G \rightarrow Q$, a positive integer

This function was implemented by VAN LUYEN LE.

`MooreComplex(G)` Inputs a simplicial group G and returns its Moore complex as a G -complex.

This function was implemented by VAN LUYEN LE.

`ChainComplexOfSimplicialGroup(G)` Inputs a simplicial group G and returns the cellular chain complex C of a C

This function was implemented by VAN LUYEN LE.

`SimplicialGroupMap(f)` Inputs a homomorphism $f : G \rightarrow Q$ of simplicial groups. The function returns an induced

This function was implemented by VAN LUYEN LE.

`HomotopyGroup(G, n)` Inputs a simplicial group G and a positive integer n . The integer n must be less than the length

Representation of elements in the bar resolution For a group G we denote by $B_n(G)$ the free $\mathbb{Z}G$ -modu

We represent a word

$$w = h_1 \cdot [g_{11} | g_{12} | \dots | g_{1n}] - h_2 \cdot [g_{21} | g_{22} | \dots | g_{2n}] + \dots + h_k \cdot [g_{k1} | g_{k2} | \dots | g_{kn}]$$

in $B_n(G)$ as a list of lists:

$$[[+1, h_1, g_{11}, g_{12}, \dots, g_{1n}], [-1, h_2, g_{21}, g_{22}, \dots, g_{2n}] + \dots + [+1, h_k, g_{k1}, g_{k2}, \dots, g_{kn}].$$

`BarResolutionBoundary(w)` This function inputs a word w in the bar resolution module $B_n(G)$ and returns its ima

This function was implemented by VAN LUYEN LE.

`BarResolutionHomotopy(w)` This function inputs a word w in the bar resolution module $B_n(G)$ and returns its ima

This function is currently being implemented by VAN LUYEN LE.

Representation of elements in the bar complex For a group G we denote by $BC_n(G)$ the free abelian group

We represent a word

$$w = [g_{11} | g_{12} | \dots | g_{1n}] - [g_{21} | g_{22} | \dots | g_{2n}] + \dots + [g_{k1} | g_{k2} | \dots | g_{kn}]$$

in $BC_n(G)$ as a list of lists:

$$[[+1, g_{11}, g_{12}, \dots, g_{1n}], [-1, g_{21}, g_{22}, \dots, g_{2n}] + \dots + [+1, g_{k1}, g_{k2}, \dots, g_{kn}].$$

`BarComplexBoundary(w)` This function inputs a word w in the n -th term of the bar complex $BC_n(G)$ and returns its

This function was implemented by VAN LUYEN LE.

`BarResolutionEquivalence(R)` This function inputs a free ZG -resolution R . It returns a component object HE wi

$$equiv(n, -): B_n(G) \rightarrow B_{n+1}(G)$$

satisfying $w - \psi(\phi(w)) = d(n+1, equiv(n, w)) + equiv(n-1, d(n, w))$. where $d(n, -): B_n(G) \rightarrow B_{n-1}(G)$ is the boundar

This function was implemented by VAN LUYEN LE.

BarComplexEquivalence(R)

This function inputs a free ZG -resolution R . It first constructs the chain complex $T = TensorWithIntegers(R)$. The function returns a component object HE with components

- HE!.phi(n,w) is a function which inputs a non-negative integer n and a word w in $BC_n(G)$. It returns the image of w in T_n under a chain equivalence $\phi: BC_n(G) \rightarrow T_n$.
- HE!.psi(n,w) is a function which inputs a non-negative integer n and an element w in T_n . It returns the image of w in $BC_n(G)$ under a chain equivalence $\psi: T_n \rightarrow BC_n(G)$.
- HE!.equiv(n,w) is a function which inputs a non-negative integer n and a word w in $BC_n(G)$. It returns the image of w in $BC_{n+1}(G)$ under a homomorphism $equiv(n, -): BC_n(G) \rightarrow BC_{n+1}(G)$ satisfying

$$w - \psi(\phi(w)) = d(n+1, equiv(n, w)) + equiv(n-1, d(n, w)).$$

where $d(n, -): BC_n(G) \rightarrow BC_{n-1}(G)$ is the boundary homomorphism in the bar complex.

This function was implemented by VAN LUYEN LE.

Representation of elements in the bar cocomplex

For a group G we denote by $BC^n(G)$ the free abelian group with basis the lists $[g_1|g_2|\dots|g_n]$ where the g_i range over G .

We represent a word

$$w = [g_{11}|g_{12}|\dots|g_{1n}] - [g_{21}|g_{22}|\dots|g_{2n}] + \dots + [g_{k1}|g_{k2}|\dots|g_{kn}]$$

in $BC^n(G)$ as a list of lists:

$$[[+1, g_{11}, g_{12}, \dots, g_{1n}], [-1, g_{21}, g_{22}, \dots, g_{2n}] + \dots + [+1, g_{k1}, g_{k2}, \dots, g_{kn}].$$

BarCocomplexCoboundary(w)

This function inputs a word w in the n -th term of the bar cocomplex $BC^n(G)$ and returns its image under the coboundary homomorphism $d^n: BC^n(G) \rightarrow BC^{n+1}(G)$ in the bar cocomplex.

This function was implemented by VAN LUYEN LE.

Chapter 21

Coxeter diagrams and graphs of groups

`CoxeterDiagramComponents(D)` Inputs a Coxeter diagram D and returns a list $[D_1, \dots, D_d]$ of the maximal connected components of D .

`CoxeterDiagramDegree(D, v)` Inputs a Coxeter diagram D and vertex v . It returns the degree of v (i.e. the number of edges incident to v).

`CoxeterDiagramDisplay(D)` `CoxeterDiagramDisplay(D, "web browser")` Inputs a Coxeter diagram D and displays it in a web browser.

`CoxeterDiagramFpArtinGroup(D)` Inputs a Coxeter diagram D and returns the corresponding finitely presented Artin group.

`CoxeterDiagramFpCoxeterGroup(D)` Inputs a Coxeter diagram D and returns the corresponding finitely presented Coxeter group.

`CoxeterDiagramIsSpherical(D)` Inputs a Coxeter diagram D and returns "true" if the associated Coxeter group is spherical.

`CoxeterDiagramMatrix(D)` Inputs a Coxeter diagram D and returns a matrix representation of it. The matrix is given by (m_{ij}) where m_{ij} is the order of $s_i s_j$.

`CoxeterSubDiagram(D, V)` Inputs a Coxeter diagram D and a subset V of its vertices. It returns the full sub-diagram with vertices V .

`CoxeterDiagramVertices(D)` Inputs a Coxeter diagram D and returns its set of vertices.

`EvenSubgroup(G)` Inputs a group G and returns a subgroup G^+ . The subgroup is that generated by all products xy where $x, y \in G$.

`GraphOfGroupsDisplay(D)` `GraphOfGroupsDisplay(D, "web browser")` Inputs a graph of groups D and displays it in a web browser.

`GraphOfResolutions(D, n)` Inputs a graph of groups D and a positive integer n . It returns a graph of resolutions of D of order n .

`GraphOfGroups(D)` Inputs a graph of resolutions D and returns the corresponding graph of groups.

`GraphOfResolutionsDisplay(D)` Inputs a graph of resolutions D and displays it as a .gif file. It uses the Mozilla browser.

`GraphOfGroupsTest(D)` Inputs an object D and tries to test whether it is a Graph of Groups. However, it DOES NOT work.

`TreeOfGroupsToContractibleGcomplex(D, G)` Inputs a graph of groups D which is a tree, and also inputs the finitely presented Coxeter group G .

`TreeOfResolutionsToContractibleGcomplex(D, G)` Inputs a graph of resolutions D which is a tree, and also inputs the finitely presented Coxeter group G .

#

Chapter 22

Torsion subcomplexes

The torsion subcomplexes subpackage has been conceived and implemented by ALEXANDER D. RAHM.

`IsPnormal(G, p)` Inputs a finite group G and a prime p . Checks if the group G is p -normal for the prime p . Zassenhaus.

`TorsionSubcomplex(groupName, p)` Inputs a cell complex with action of a group. In HAP, presently the following

"SL(2,O[-2])" , "SL(2,O[-7])" , "SL(2,O[-11])" , "SL(2,O[-19])" , "SL(2,O[-43])" , "SL(2,O[-67])" , "SL(2,O[-163])"

where the symbol $O[-m]$ stands for the ring of integers in the imaginary quadratic number field $\mathbb{Q}(\sqrt{-m})$, the latter

The function `TorsionSubcomplex` prints the cells with p -torsion in their stabilizer on the screen and returns the incidence

It is also possible to input the cell complexes

"SL(2,Z)" , "SL(3,Z)" , "PGL(3,Z[i])" , "PGL(3,Eisenstein_Integers)" , "PSL(4,Z)" , "PSL(4,Z)_b" , "PSL(4,Z)_c" ,

provided by MATHIEU DUTOUR, only there will be some warnings printed on the screen regarding the function `reduce`

`DisplayAvailableCellComplexes()`; Displays the cell complexes that are available in HAP.

`VisualizeTorsionSkeleton(groupName, p)` Executes the function `TorsionSubcomplex(groupName, p)` and visualizes

`ReduceTorsionSubcomplex(groupName, p)` This function may be applied to the cell complexes for which the function

Chapter 23

Simplicial Complexes

`Homology(T, n)` `Homology(T)` Inputs a pure cubical complex, or cubical complex, or simplicial complex T and a non-negative integer n . It returns the n -th homology group of T .

`RipsHomology(G, n)` `RipsHomology(G, n, p)` Inputs a graph G , a non-negative integer n (and optionally a prime number p). It returns the n -th homology group of the Rips complex of G .

`Bettinnumbers(T, n)` `Bettinnumbers(T)` Inputs a pure cubical complex, or cubical complex, simplicial complex T and a non-negative integer n . It returns the n -th Betti number of T .

`ChainComplex(T)` Inputs a pure cubical complex, or cubical complex, or simplicial complex T and returns the (offset) chain complex of T .

`CechComplexOfPureCubicalComplex(T)` Inputs a d -dimensional pure cubical complex T and returns a simplicial complex.

`PureComplexToSimplicialComplex(T, k)` Inputs either a d -dimensional pure cubical complex T or a d -dimensional cubical complex T and a non-negative integer k . It returns the k -skeleton of T .

`RipsChainComplex(G, n)` Inputs a graph G and a non-negative integer n . It returns $n + 1$ terms of a chain complex of the Rips complex of G .

`VectorsToSymmetricMatrix(M)` `VectorsToSymmetricMatrix(M, distance)` Inputs a matrix M of rational numbers and a non-negative integer $distance$. It returns a symmetric matrix.

`EulerCharacteristic(T)` Inputs a pure cubical complex, or cubical complex, or simplicial complex T and returns the Euler characteristic of T .

`MaximalSimplicesToSimplicialComplex(L)` Inputs a list L whose entries are lists of vertices representing the maximal simplices of a simplicial complex. It returns the simplicial complex.

`SkeletonOfSimplicialComplex(S, k)` Inputs a simplicial complex S and a positive integer k less than or equal to the dimension of S . It returns the k -skeleton of S .

`GraphOfSimplicialComplex(S)` Inputs a simplicial complex S and returns the graph of S .

`ContractibleSubcomplexOfSimplicialComplex(S)` Inputs a simplicial complex S and returns a (probably maximal) contractible subcomplex of S .

`PathComponentsOfSimplicialComplex(S, n)` Inputs a simplicial complex S and a nonnegative integer n . If $n = 0$ it returns the number of path components of S . If $n > 0$ it returns the number of path components of the n -skeleton of S .

`QuillenComplex(G)` Inputs a finite group G and returns, as a simplicial complex, the order complex of the poset of proper subgroups of G .

`SymmetricMatrixToIncidenceMatrix(S, t)` `SymmetricMatrixToIncidenceMatrix(S, t, d)` Inputs a symmetric 0/1 matrix M and a non-negative integer t (and optionally a non-negative integer d). It returns the incidence matrix of the t -skeleton of the simplicial complex defined by M .

`IncidenceMatrixToGraph(M)` Inputs a symmetric 0/1 matrix M . It returns the graph with one vertex for each row of M and edges between vertices i and j if $M_{ij} = 1$.

`CayleyGraphOfGroup(G, A)` Inputs a group G and a set A of generators. It returns the Cayley graph.

`PathComponentsOfGraph(G, n)` Inputs a graph G and a nonnegative integer n . If $n = 0$ it returns the number of path components of G . If $n > 0$ it returns the number of path components of the n -skeleton of G .

`ContractGraph(G)` Inputs a graph G and tries to remove vertices and edges to produce a smaller graph G' such that G and G' have the same homology.

`GraphDisplay(G)` This function uses `GraphViz` software to display a graph G .

`SimplicialMap(K, L, f)` `SimplicialMapNC(K, L, f)` Inputs simplicial complexes K, L and a function $f: K \rightarrow L$. It returns a simplicial map $f: K \rightarrow L$.

`ChainMapOfSimplicialMap(f)` Inputs a simplicial map $f: K \rightarrow L$ and returns the corresponding chain map $C_*(f): C_*(K) \rightarrow C_*(L)$.

`SimplicialNerveOfGraph(G, d)` Inputs a graph G and returns a d -dimensional simplicial complex K whose 1-skeleton is G .

Chapter 24

Cubical Complexes

`ArrayToPureCubicalComplex(A,n)` Inputs an integer array A of dimension d and an integer n . It returns a d -dimensional pure cubical complex.

`PureCubicalComplex(A,n)` Inputs a binary array A of dimension d . It returns the corresponding d -dimensional pure cubical complex.

`PureCubicalComplexIntersection(S,T)` Inputs two pure cubical complexes with common dimension and array size. It returns their intersection.

`PureCubicalComplexUnion(S,T)` Inputs two pure cubical complexes with common dimension and array size. It returns their union.

`PureCubicalComplexDifference(S,T)` Inputs two pure cubical complexes with common dimension and array size. It returns their difference.

`ReadImageAsPureCubicalComplex("file.png",n)` Reads an image file ("file.png", "file.eps", "file.bmp" etc) and returns a pure cubical complex.

`ReadLinkImageAsPureCubicalComplex("file.png")` Reads a link image file and returns a pure cubical complex.

`ReadImageSequenceAsPureCubicalComplex("directory",n)` Reads the name of a directory containing a sequence of image files and returns a pure cubical complex.

`Size(T)` This returns the number of non-zero entries in the binary array of the cubical complex, or pure cubical complex T .

`Dimension(T)` This returns the dimension of the cubical complex, or pure cubical complex T .

`WritePureCubicalComplexAsImage(T,"filename","ext")` Inputs a 2-dimensional pure cubical complex T , and a filename and extension. It writes the image to the file.

`ViewPureCubicalComplex(T)` `ViewPureCubicalComplex(T,"mozilla")` Inputs a 2-dimensional pure cubical complex T . It displays the image in a browser window.

`Homology(T,n)` `Homology(T)` Inputs a pure cubical complex, or cubical complex, or simplicial complex T and a non-negative integer n . It returns the n -th homology group.

`Bettinnumbers(T,n)` `Bettinnumbers(T)` Inputs a pure cubical complex, or cubical complex, simplicial complex or simplicial set T and a non-negative integer n . It returns the n -th Betti number.

`DirectProductOfPureCubicalComplexes(M,N)` Inputs two pure cubical complexes M,N and returns their direct product.

`SuspensionOfPureCubicalComplex(M)` Inputs a pure cubical complex M and returns a pure cubical complex with one higher dimension.

`EulerCharacteristic(T)` Inputs a pure cubical complex, or cubical complex, or simplicial complex T and returns the Euler characteristic.

`PathComponentOfPureCubicalComplex(T,n)` Inputs a pure cubical complex T and an integer n in the range $1, \dots, \dim T$. It returns a path component.

`ChainComplex(T)` Inputs a pure cubical complex, or cubical complex, or simplicial complex T and returns the (ordered) chain complex.

`ChainComplexOfPair(T,S)` Inputs a pure cubical complex or cubical complex T and subcomplex S . It returns the chain complex of the pair.

`ExcisedPureCubicalPair(T,S)` Inputs a pure cubical complex T and subcomplex S . It returns the pair $[T \setminus \text{int} S, S]$.

`ChainInclusionOfPureCubicalPair(S,T)` Inputs a pure cubical complex T and subcomplex S . It returns the chain inclusion.

`ChainMapOfPureCubicalPairs(M,S,N,T)` Inputs a pure cubical complex N and subcomplexes M, T and S in T . It returns a chain map.

`ContractPureCubicalComplex(T)` Inputs a pure cubical complex T of dimension d and removes d -dimensional cells.

`ContractedComplex(T)` Inputs a pure cubical complex T and returns a structural copy of the complex obtained from `ContractPureCubicalComplex(T)`.

`ZigZagContractedPureCubicalComplex(T)` Inputs a pure cubical complex T and returns a homotopy equivalent cubical complex.

`ContractCubicalComplex(T)` Inputs a cubical complex T and removes cells without changing the homotopy type.

`DVFRducedCubicalComplex(T)` Inputs a cubical complex T and returns a non-regular cubical complex R by constant deformation.

`SkeletonOfCubicalComplex(T,n)` Inputs a cubical complex, or pure cubical complex T and positive integer n . It returns the n -skeleton.

`ContractibleSubcomplexOfPureCubicalComplex(T)` Inputs a pure cubical complex T and returns a maximal contractible subcomplex.

`AcyclicSubcomplexOfPureCubicalComplex(T)` Inputs a pure cubical complex T and returns a (not necessarily contractible) acyclic subcomplex.

`HomotopyEquivalentMaximalPureCubicalSubcomplex(T,S)` Inputs a pure cubical complex T together with a subcomplex S . It returns a maximal subcomplex homotopy equivalent to S .

`HomotopyEquivalentMinimalPureCubicalSubcomplex(T,S)` Inputs a pure cubical complex T together with a subcomplex S . It returns a minimal subcomplex homotopy equivalent to S .

`BoundaryOfPureCubicalComplex(T)` Inputs a pure cubical complex T and returns its boundary as a pure cubical complex.

`SingularitiesOfPureCubicalComplex(T,radius,tolerance)` Inputs a pure cubical complex T together with a radius and tolerance. It returns the singularities.

`ThickenedPureCubicalComplex(T)` Inputs a pure cubical complex T and returns a pure cubical complex S . If a euclidean neighborhood deformation retract.

`CropPureCubicalComplex(T)` Inputs a pure cubical complex T and returns a pure cubical complex S obtained from T by cropping.

`BoundingPureCubicalComplex(T)` Inputs a pure cubical complex T and returns a contractible pure cubical complex bounding T .

`MorseFiltration(M,i,t,bool)` `MorseFiltration(M,i,t)` Inputs a pure cubical complex M of dimension d , an integer i , a real number t , and a boolean. It returns the Morse filtration.

`ComplementOfPureCubicalComplex(T)` Inputs a pure cubical complex T and returns a pure cubical complex S . A euclidean neighborhood deformation retract.

`PureCubicalComplexToTextFile(file,M)` Inputs a pure cubical complex M and a string containing the address of a text file. It writes the complex to the file.

Chapter 25

Regular CW-Spaces

`SimplicialComplexToRegularCWSpace(K)` Inputs a simplicial complex K and returns the corresponding regular CW-space.

`CubicalComplexToRegularCWSpace(K)` Inputs a pure cubical complex (or cubical complex) K and returns the corresponding regular CW-space.

`CriticalCellsOfRegularCWSpace(Y)` Inputs a regular CW-space Y and returns the critical cells of Y with respect to a given metric.

`ChainComplex(Y)` Inputs a regular CW-space Y and returns the cellular chain complex of a CW-space W whose cells are the critical cells of Y .

`ChainComplexOfRegularCWSpace(Y)` Inputs a regular CW-space Y and returns the cellular chain complex of Y .

`FundamentalGroup(Y)` `FundamentalGroup(Y, n)` Inputs a regular CW-space Y and, optionally, the number of generators n .

Chapter 26

Commutative diagrams and abstract categories

COMMUTATIVE DIAGRAMS

HomomorphismChainToCommutativeDiagram(H) Inputs a list $H = [h_1, h_2, \dots, h_n]$ of mappings such that the composition is the identity.
NormalSeriesToQuotientDiagram(L) NormalSeriesToQuotientDiagram(L, M) Inputs an increasing (or decreasing) normal series L of a group G and a normal subgroup M of G .
NerveOfCommutativeDiagram(D) Inputs a commutative diagram D and returns the commutative diagram ND corresponding to D .
GroupHomologyOfCommutativeDiagram(D, n) GroupHomologyOfCommutativeDiagram(D, n, prime) GroupHomologyOfCommutativeDiagram(D, n, prime, prime) Inputs a commutative diagram D and returns the n -th homology group of D .
PersistentHomologyOfCommutativeDiagramOfPGroups(D, n) Inputs a commutative diagram D of finite p -groups and returns the persistent homology of D .

ABSTRACT CATEGORIES

CategoricalEnrichment(X, Name) Inputs a structure X such as a group or group homomorphism, together with a name $Name$ for the category.
IdentityArrow(X) Inputs an object X in some category, and returns the identity arrow on the object X .
InitialArrow(X) Inputs an object X in some category, and returns the arrow from the initial object in the category to X .
TerminalArrow(X) Inputs an object X in some category, and returns the arrow from X to the terminal object in the category.
HasInitialObject(Name) Inputs the name of a category and returns true or false depending on whether the category has an initial object.
HasTerminalObject(Name) Inputs the name of a category and returns true or false depending on whether the category has a terminal object.
Source(f) Inputs an arrow f in some category, and returns its source.
Target(f) Inputs an arrow f in some category, and returns its target.
CategoryName(X) Inputs an object or arrow X in some category, and returns the name of the category.
"*", "=", "+", "-" Composition of suitable arrows f, g is given by $f * g$ when the source of f equals the target of g .
Object(X) Inputs an object X in some category, and returns the GAP structure Y such that $X = CategoricalEnrichment(Y)$.
Mapping(X) Inputs an arrow f in some category, and returns the GAP structure Y such that $f = CategoricalEnrichment(Y)$.
IsCategoryObject(X) Inputs X and returns true if X is an object in some category.
IsCategoryArrow(X) Inputs X and returns true if X is an arrow in some category.

Chapter 27

Arrays and Pseudo lists

`Array(A, f)` Inputs an array A and a function f . It returns the the array obtained by applying f to each entry of A (and

`PermuteArray(A, f)` Inputs an array A of dimension d and a permutation f of degree at most d . It returns the array

`ArrayDimension(A)` Inputs an array A and returns its dimension.

`ArrayDimensions(A)` Inputs an array A and returns its dimensions.

`ArraySum(A)` Inputs an array A and returns the sum of its entries.

`ArrayValue(A, x)` Inputs an array A and a coordinate vector x . It returns the value of the entry in A with coordinate

`ArrayValueFunctions(d)` Inputs a positive integer d and returns an efficient version of the function `ArrayValue` for

`ArrayAssign(A, x, n)` Inputs an array A and a coordinate vector x and an integer n . It sets the entry of A with coord

`ArrayAssignFunctions(d)` Inputs a positive integer d and returns an efficient version of the function `ArrayAssign`

`ArrayIterate(d)` Inputs a positive integer d and returns a function `ArrayIt(Dimensions, f)`. This function inputs a li

`BinaryArrayToTextFile(file, A)` Inputs a string containing the address of a file, and an array A of 0s and 1s. The

`FrameArray(A)` Inputs an array A and returns the array obtained by appending a 0 to the beginning and end of each

`UnframeArray(A)` Inputs an array A and returns the array obtained by removing the first and last entry in each "row"

`Add(L, x)` Let L be a pseudo list of length n , and x an object compatible with the entries in L . If x is not in L then thi

`Append(L, K)` Let L be a pseudo list and K a list whose objects are compatible with those in L . This operation applie

`ListToPseudoList(L)` Inputs a list L and returns the pseudo list representation of L .

Chapter 29

Parallel Computation - Extra Functions

`ChildFunction("function(arg);",s)` This runs the GAP function "function(arg);" on a child process accessed by `s`.

`ChildRead(s)` This returns, as a string, the output of the last application of `ChildFunction("function(arg);",s)`.

`ChildReadEval(s)` This returns, as an evaluated string, the output of the last application of `ChildFunction("function(arg);",s)`.

`ParallelList(I,fn,L)` Inputs a list I , a function fn such that $fn(x)$ is defined for all x in I , and a list of children L .

Chapter 30

Some functions for accessing basic data

`BoundaryMap(C)` Inputs a resolution, chain complex or cochain complex C and returns the function $C!.boundary$.

`BoundaryMatrix(C, n)` Inputs a chain or cochain complex C and integer $n > 0$. It returns the n -th boundary map of C .

`Dimension(C)`

`Dimension(M)` Inputs a resolution, chain complex or cochain complex C and returns the function $C!.dimension$.

`EvaluateProperty(X, "name")` Inputs a component object X (such as a ZG -resolution or chain map) and a string `name`.

`GroupOfResolution(R)` Inputs a ZG -resolution R and returns the group G .

`Length(R)` Inputs a resolution R and returns its length (i.e. the number of terms of R that HAP has computed).

`Map(f)` Inputs a chain map, or cochain map or equivariant chain map f and returns the mapping function (as opposed to the map itself).

`Source(f)` Inputs a chain map, or cochain map, or equivariant chain map, or FpG -module homomorphism f and returns the source object.

`Target(f)` Inputs a chain map, or cochain map, or equivariant chain map, or FpG -module homomorphism f and returns the target object.

Chapter 31

Miscellaneous

`SL2Z(p)` `SL2Z(1/m)` Inputs a prime p or the reciprocal $1/m$ of a square free integer m . In the first case the function returns a list of all subgroups of $SL_2(\mathbb{Z})$ of index p . In the second case it returns a list of all subgroups of $SL_2(\mathbb{Z})$ of index m .

`BigStepLCS(G,n)` Inputs a group G and a positive integer n . It returns a subseries $G = L_1 > L_2 > \dots > L_k = 1$ of the lower central series of G such that $[L_i, L_i] = 1$ for all i .

`Classify(L,Inv)` Inputs a list of objects L and a function Inv which computes an invariant of each object. It returns a list of objects C such that $Inv(C) = Inv(L)$ for all $C \in C$.

`RefineClassification(C,Inv)` Inputs a list $C := Classify(L, OldInv)$ and returns a refined classification according to the function Inv .

`Compose(f,g)` Inputs two FpG -module homomorphisms $f : M \rightarrow N$ and $g : L \rightarrow M$ with $Source(f) = Target(g)$. It returns the composition $f \circ g$.

`HAPcopyright()` This function provides details of HAP'S GNU public copyright licence.

`IsLieAlgebraHomomorphism(f)` Inputs an object f and returns true if f is a homomorphism $f : A \rightarrow B$ of Lie algebras.

`IsSuperperfect(G)` Inputs a group G and returns "true" if both the first and second integral homology of G is trivial.

`MakeHAPManual()` This function creates the manual for HAP from an XML file.

`PermToMatrixGroup(G,n)` Inputs a permutation group G and its degree n . Returns a bijective homomorphism $f : G \rightarrow GL_n(\mathbb{Z})$.

`SolutionsMatDestructive(M,B)` Inputs an $m \times n$ matrix M and a $k \times n$ matrix B over a field. It returns a $k \times m$ matrix X such that $MX = B$.

`LinearHomomorphismsPersistenceMat(L)` Inputs a composable sequence L of vector space homomorphisms. It returns a matrix M such that $M_{ij} = \dim(L_i) - \dim(L_{i+1})$.

`NormalSeriesToQuotientHomomorphisms(L)` Inputs an (increasing or decreasing) chain L of normal subgroups of a group G . It returns a list of homomorphisms $f_i : G/L_i \rightarrow G/L_{i+1}$.

`TestHap()` This runs a representative sample of HAP functions and checks to see that they produce the correct output.

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